Policy, Planning, and Research

WORKING PAPERS

Public Economics

Country Economics Department
and Western Africa Department
The World Bank
May 1989
WPS 198

Equity in Unequal Deductions

Implications of Income Tax Rules in Ghana and Nigeria

Chad Leechor
and
Robert Warner

In many African countries, the size of a taxpayer's personal deduction increases with his income. Does this rule give the rich more of a tax break than the poor? Is a standard (fixed) deduction needed to allow for progressivity? The answers may surprise you.

Policy, Planning, and Research

WORKING PAPERS

Public Economics

At first glance, allowing the personal deduction on income tax to increase as the taxpayer's income increases appears to give larger tax breaks to the rich than to the poor. On closer examination, this notion turns out to be false. As the authors of this paper learned, each tax system with "income-dependent tax deductions" (IDTDs) is fully equivalent to a particular conventional progressive tax system with standard deductions.

Consider a given conventional tax schedule that has standard deductions and progressive tax rates. Suppose that you add a new rule to this system that provides an additional deduction equal to 10 percent of the taxpayer's income. This single reform measure has the same effect as a "liberalization" package consisting of (a) an increase of about 10 percent in standard deductions, (b) an enlargement of about 10 percent in all tax brackets, and (c) a reduction of about 10 percent on all marginal tax rates. The full equivalence of the two options may not be obvious — it was not to the authors.

In other words, a tax system with IDTDs is not less equitable than a conventional system with standard deductions. They are equivalent. The liberalization package in the example is typical in 1980s' tax reforms yet equity has not emerged as an issue. An equivalent tax reform option consisting of an IDTD rule, therefore, should not be a cause for concern. Substituting an IDTD rule for a liberalization package would leave the tax system effectively unchanged, although the two sets of tax rules appear to be different.

Should a country contemplating liberalization of the tax schedules simply adopt an IDTD? The answer is not easy. An IDTD is administratively simple, but its logic is not immediately transparent. It could be seen as inequitable, and it could be misused. The fixed percentage deduction could be relaxed, resulting in an unrestricted deduction. The IDTD could also be confined to one category of taxpayers, resulting in a true inequity — as with discrimination against the self-employed in some countries.

These caveats do not necessarily justify replacing an existing IDTD with an equivalent liberalization package. A taxpaying public long accustomed to income-determined allowances may vigorously oppose such a change. Even in an imperfect IDTD system, it may be strategically preferable to correct the aberrations rather than eliminate the IDTDs and risk a tax revolt.

One implication for comparative tax research is that the tax schedule of a country that uses IDTDs should not be compared directly with a conventional tax schedule in another country. Existing cross-country work on tax deductions and marginal tax rates generally fails to recognize that IDTDs invalidate a straightforward comparison. To make the two systems comparable, a transformation like the one suggested in the paper is needed.

This paper is a product of thePublic Economics Division, Country Economics Department and the Country Operations Division, Western Africa Department. Copies are available free from the World Bank, 1818 H Street NW, Washington DC 20433. Please contact Ann Bhalla, room N10-061, extension 60359.

EQUITY IN UNEQUAL DEDUCTIONS:
Implications of Income Tax Rules in Ghana and Nigeria

Tables of Contents

Bela Balassa, Javad Khalilzadeh-Shirazi, Steve Mink, William McCleary, Vikram Nehru, Zmarak Shalizi and Lyn Squire provided valuable comments on an earlier draft. Reza Firuzabadi produced the elegant diagrams. Carlina Jones typed the text with patience and good cheer.

I. INTRODUCTION

1. In many African countries, the amount of personal deduction for income tax purposes increases with the taxpayer's income. (This practice henceforth will be referred to as "income-dependent tax deductions" or IDTDs.) This rule is unconventional in many respects. First, contrary to the common practice across countries, it allows the amount of personal deduction to vary from one taxpayer to another. Second, the variation in deductions does not depend on objective indicators of needs, such as the number of dependents in a household. Third, whereas a fixed or standard deduction is meant to provide full or partial tax relief to the poorest income group, the IDTD rule provides tax reductions for everybody across the board.

2. To a tax analyst accustomed to conventional rules, the dependency of personal deduction on income appears to be a dubious proposition. To begin with, there seems to be no point in providing very large tax relief for individuals with very high income, as is the case under the IDTD rule. An inverse relationship between income and personal deduction would appear more desirable than a direct relationship, for at least more revenue could be raised with no objectionable hardship created. Moreover, when income-dependent tax deductions are combined with a progressive rate structure, the distribution of tax savings across individuals becomes highly inequitable. As income rises, both personal deductions and marginal tax rates increase, and therefore tax savings are amplified.

3. These concerns about IDTDs are rooted in traditional income tax analysis. At the heart of the concerns is the notion that a particular provision implies more benefits for the rich than the rest of society. No

specific conflict with horizontal or vertical equity is cited. A very similar issue well-known in the income tax literature is the comparison of tax deductions and tax credit.[1] It is typically demonstrated that tax credit is a better instrument of tax relief for the poor. According to the standard argument, the deduction method allows tax savings to rise in proportion to marginal tax rates whereas the tax credit method creates no such discrepancy. An illustration of this argument is given in the Annex.

4. The objections against IDTDs are based on relatively stringent equity conditions Less severe requirements are generally met by the IDTD rule. Consider, for example, the vertical equity test. It can be shown that the effective tax rate (defined as total taxes paid divided by gross income) under IDTDs does not fall as income increases. Indeed, any desired degree of progressivity can be achieved through appropriate graduation of tax rates. Moreover, IDTDs create no tax discrepancy for individuals with the same level of income and therefore satisfies the horizontal equity requirement. Finally, IDTDs may be used to exclude the poor of any chosen income range from the tax net. This objective can be accomplished by choosing the appropriate level of the minimum deduction available to all taxpayers.

5. Nevertheless, to the traditional tax analyst, a tax system with a fixed or standard tax deduction (STD) is superior, at least on equity grounds, to one with IDTDs. Under STDs, there is a limit placed on the amount of tax relief claimed by those with high income. No such limit exists under IDTDs. Moreover, STDs do not produce sharply divergent tax savings as do IDTDs. A tax reform involving a replacement of IDTDs by STDs

---

1/ See, for example, Lewis (1984), pp. 65-76.

would therefore be desirable. Such a reform would improve equity and, if desired, raise more revenue through appropriate choice of the deduction level.

6. These conclusions are incorrect, however. As the analysis below will demonstrate, the traditional tax analyst is misguided in his assessments. IDTDs are in fact equivalent to STDs. There exists, for example, a perfectly conventional tax system (with STDs among others) that produces for all taxpayers exactly the same tax liabilities as the existing tax system of Ghana or Nigeria. Tax rates and brackets associated with the equivalent conventional systems will differ from the existing ones, however. These results are somewhat surprising and may serve as a reminder against the tendency to make quick judgements on unconventional practices and against the inclination to fall back on the familiar.

II. EXAMPLES OF UNEQUAL DEDUCTIONS

7. Systematic use of income-dependent tax deductions (IDTDs) is found in Ghana and Nigeria. On the surface, deduction rules in the two countries appear to be very different. Nigeria has a relatively simple rule in which deduction is the sum of a fixed amount (of dependent allowances) and a percentage of income. The relationship is immediately discernible:

$$D = \alpha + \beta \cdot Y \qquad (1)$$

where D = personal deduction

Y = income

8. The case of Ghana is more complex. Personal relief consists of a standard deduction under the tax schedule and tax-free allowances, which vary across individuals. The amount of tax-free allowances permissible by

law, however, is systematically related to income. And hence the general relationship in (1) also holds for the Ghanaian tax system. Figure 1 shows the particular functional relations for the two countries.

A. The Case of Ghana

9. The Ghanaian tax code recognizes two distinct forms of earned income: basic salary and essential allowances. The basic salary is subject to tax and serves as a benchmark for a variety of financial calculations including pension-fund contribution and social security taxes. The allowances are meant to defray the taxpayer's living expenses and are not taxable. Only salaried workers may claim tax-free allowances, however; the self-employed may deduct business expenses, but once income is determined, no further allowances are granted.[2] The tax schedule looks very much like a conventional system, with a standard deduction and progressive tax rates, but it applies only to basic salaries. A taxpayer is entitled to deduct from his total income (a) all the allowances and (b) a standard deduction given in the tax schedule.

10. During the calendar year 1988, the following tax schedule was in effect:

Bracket (C)[3]	Tax Rate (%)
First 24,000[4]	Nil
Next 30,000	5
Next 30,000	10
Next 225,000	20
Next 225,000	30
Next 225,000	40
Next 225,000	50
Over 984,000	55

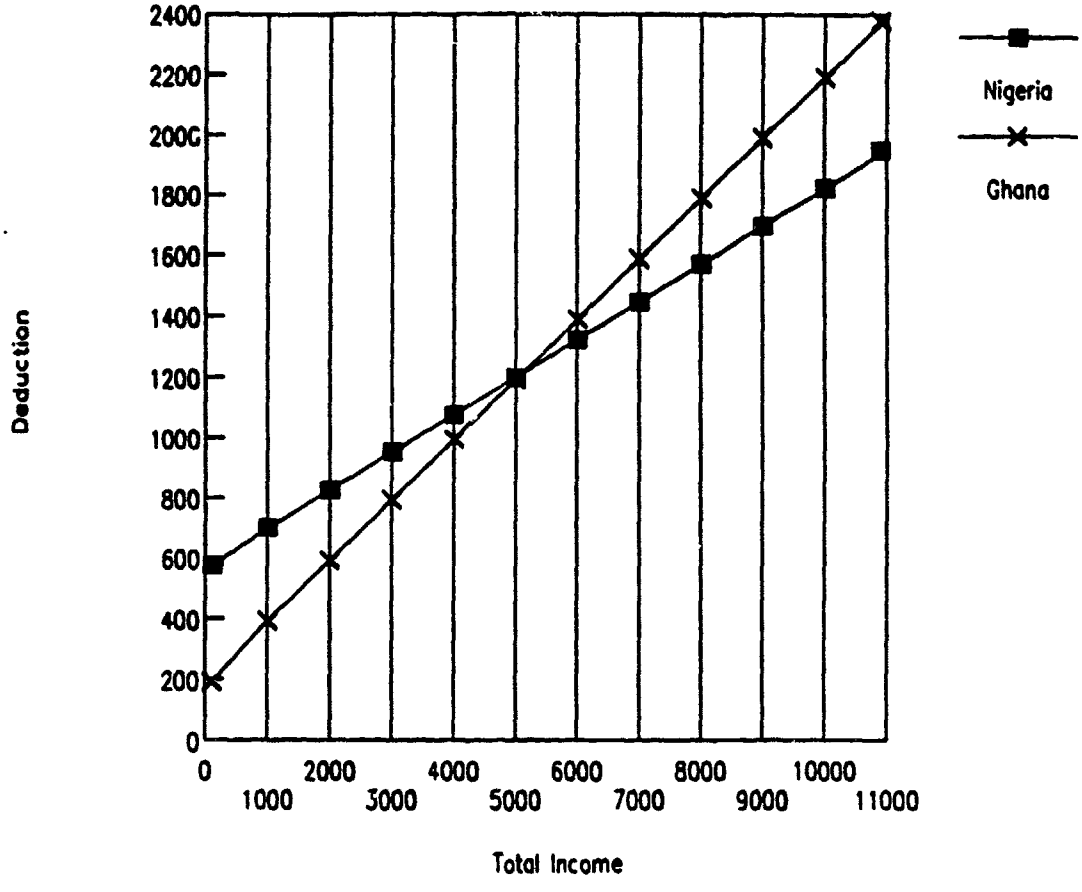
2/ This rule discriminates against the self-employed and violates the principle of horizontal equity. The current impact of the discrimination may not be significant, however, since there is little control at present over the types and scale of business expenses claimed. As the audit capacity improves, however, such a rule would become a more serious cause for concern.

3/ The average official (auction) exchange rate in 1988 was C 205/US$

4/ The amount could in principle be adjusted for dependents, but in practice no adjustments were allowed.

Figure 1

Ghana and Nigeria
Income - Dependent Tax Deduction



Note: Local Currencies have been converted into a common denomination (US$)

Sources:
1. Ghana: Internal Revenue Service
2. Nigeria: Federal Inland Revenue Department
3. Staff Calculations

11. In addition, the following guidelines on tax-free compensation were used:

(1) housing allowance up to 20% of basic salary
(2) bonus, up to 15% of basic salary
(3) transport allowance, no limit in 1988[5]
(4) canteen allowance, no limit in 1988[5]
(5) car maintenance allowance, up to C 30,000 p.a.
(6) leave allowance, up to C 1,500 p.a.
(7) night duty allowance, no limit.

The amount of allowances received by an employee thus depends on his basic salary (items 1 and 2) and on the types of fixed allowances granted. For instance, in 1988, low-level civil servants received about C 35,000 in tax-free allowances, whereas top-level officials were given about C 120,000.

12. The relationship between tax-free allowances and basic salaries may be expressed as follows:

$$A = \gamma + \delta \cdot B \qquad (2)$$

where
A = tax-free allowances
B = basic salary
γ = total fixed allowances received
δ = housing and bonus allowances expressed as a fraction of basic salary

For the calendar year 1988, the values for γ and δ may be conservatively[6] estimated as follows:

$$\gamma = C\ 18{,}300 \qquad (3)$$
$$\delta = 0.20 \qquad (4)$$

The estimate of γ is based on the combined limits of transport, canteen and leave allowances permitted in 1987. The estimate for δ is based on

---

5/ Until December 1987, cedi limits had been given on transport and canteen allowances. The PNDC Law 193 (Income Tax Amendment Number 3, 1987) removed the limits, but placed a discretionary power on the Commissioner to rule out "excessive" allowances.

6/ "Conservative" implies a tendency toward giving low values compared to the most probable outcome.

housing allowances alone, with bonuses omitted (since no data are available).

13. With the foregoing result, the relationship between personal deduction and total income may now be derived. Personal deduction consists of two parts: first the tax-free allowances and second the standard deduction, which may be written as follows:

$$D = A + s \qquad (5)$$

where D = total personal deduction
A = tax-free allowances
s = standard deduction under the tax schedule

14. Since total income is the sum of basic salary and tax-free allowances:

$$Y = A + B; \qquad (6)$$

deduction may be expressed in terms of total income:

$$D = \alpha_G + \beta_G \cdot Y \qquad (7)$$

where $\alpha_G = s + [\gamma/(1+\delta)]$

and $\beta_G = \delta/(1+\delta)$

According to the tax schedule of 1988, with

$$\alpha_G = 39{,}250$$

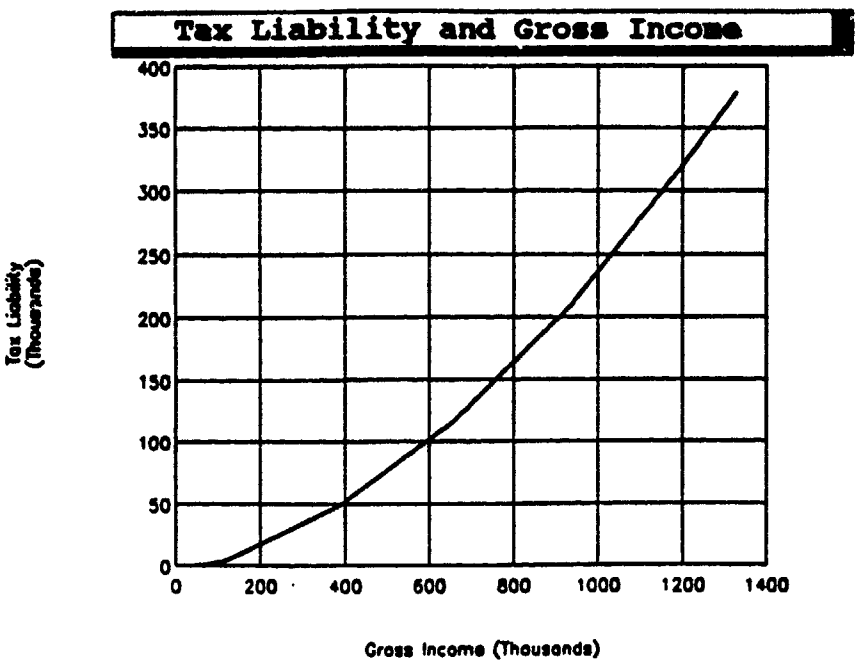
$$\beta_G = 0.167,$$

the relationship is

$$D = 39{,}250 + 0.167 \cdot Y \qquad (8)$$

15. In spite of the IDTDs, the overall tax schedule is progressive. The relationship between (total) income and tax liability is shown in Figure 2. As income rises, the tax liability becomes a larger proportion of income. The slope of a line segment connecting the origin and a point on the curve increases with the horizontal distance from the origin.

Figure 2
GHANA



B. The Case of Nigeria

16. Personal deductions under the Nigerian income tax code consist of two components. The first depends on the number of dependents in the household. The second is stated as a fraction of the taxpayer's earned income. Unlike the case of Ghana, these deductions may be claimed by the self employed and independent professionals.

17. These rules imply the following relationship between the level of personal deduction and income:

$$D = \alpha_N + \beta_N \cdot Y \qquad (9)$$

where D = personal deduction

Y = total earned income

α_N = dependent allowance

β_N = the fraction of deductible income

The value of Y vary across taxpayers while that of α_N and β_N change over time.

18. During the calendar year 1987,[7] the following tax schedule was implemented:

1. Deductions

Personal deduction was set at Naira 1,000 plus 12.5% of earned income. Dependent allowances included N500 for spouse, N400 for each child up to a maximum of four, and N600 for each dependent relative with no limit stated.

7/ The average official exchange rate in 1987 was N4.01 to the US$.

2. **Tax Rates**

Bracket (Taxable income)	Rate (%)
First N2,000	10
Next N2,000	15
Next N2,000	20
Next N2,000	25
Next N5,000	35
Next N5,000	40
Next N10,000	45
Next N10,000	50
Over N40,000	55

19. Thus, for a taxpayer with a spouse and two dependent children, personal deduction may be written as:

$$D = 2{,}300 + 0.125 * Y \qquad (10)$$

Consequently, taxable income is given by:

$$I = Y-D = -2{,}300 + 0.875.Y \qquad (11)$$

where I = Taxable Income.

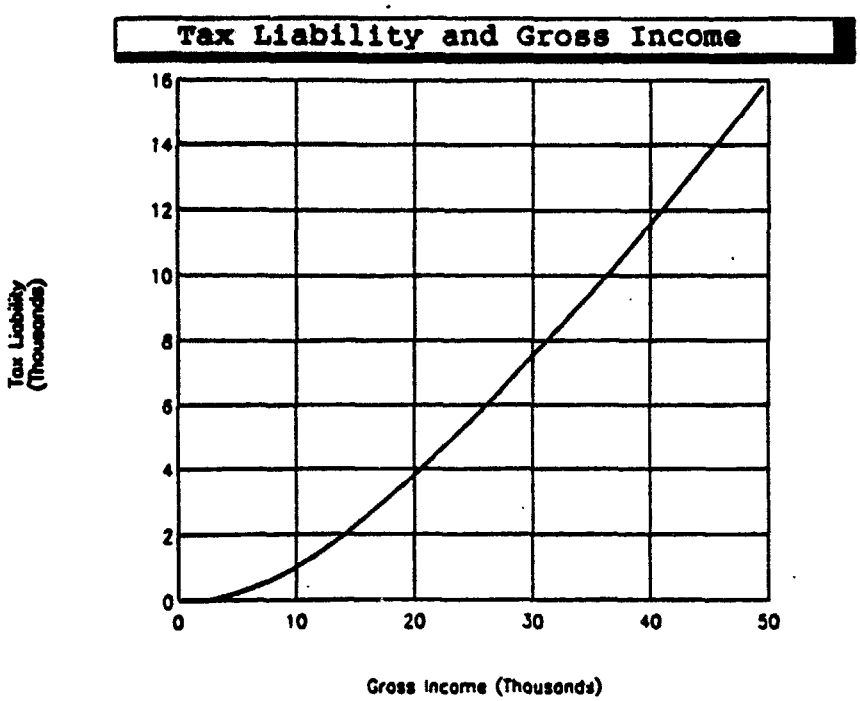
Like the case of Ghana, the overall tax burden is progressive, in spite of the IDTDs, as shown in Figure 3.

III. ANALYSIS

20. This section demonstrates a transformation of tax systems based on partial income, such as that of Ghana and Nigeria, to one based on total income. This transformation yields a new tax structure that provides considerable, and unexpected, insights into the use of IDTDs. This new structure also serves as the correct basis for comparisons with conventional tax systems elsewhere.[8]

8/ A method for comparing personal income tax schedules across countries is given in Sicat and Virmani (1988). The authors did not account for the use of IDTDs among countries in the sample, however.

Figure 3
NIGERIA



21. The following analysis rests on the realization that the information contained in a personal income tax schedule may be summarized in a compact equation. This equation expresses the relationship between a given level of income and the tax liability implied by the schedule and may be written algebraically as follows:

$$T = m_i \cdot (Y - x) - k_i \qquad (12)$$

where
- T = income tax due;
- m_i = the marginal tax rate in the ith bracket to which Y belongs;
- Y = the given level of total income
- x = fixed or standard deduction
- k_i = a constant specific to the ith bracket

The values of the various parametric constants (m_i's, x and k_i's) are derived from the given tax schedule. The constants k_i's represent an adjustment factor needed to allow for progressivity in the tax schedule and are defined as follows:

$$k_i = 0 \qquad \text{for } i = 0, 1 \qquad (13)$$

$$k_i = \sum_{j=1}^{i-1} (m_i - m_j) \cdot B_j. \qquad \text{for } i > 1 \qquad (14)$$

where
- B_j = the size of the jth tax bracket, with the <u>first</u> bracket referring to the income range to which the <u>lowest</u> non-zero marginal tax rate applies.

22. Consider, for example, the following income tax schedule:

<u>Tax Rate</u>	<u>Bracket</u>
Nil	First 10,000
10%	Next 10,000
20%	Next 10,000
30%	Above 30,000

From this schedule, the following constant values are determined:

i	m_i	B_i	k_i
0	0	10,000	0
1	10%	10,000	0
2	20%	10,000	1,000
3	30%	N.A.	3,000

Normally, the last bracket (that with the highest marginal tax rate) is open-ended and therefore infinite in size. The calculation of k_i's, as given in (14), does not require the size of the last bracket, however, and thus no indeterminancy arises. For instance, in the example above, k_3 depends only on B_1 and B_2 (each equal to 10,000) and not at all on B_3, which is open-ended.

23. The fundamental equation (12) is useful in a variety of applications. The principal use in this section is to convert the Ghanaian and the Nigerian tax systems into an equivalent, but more conventional and familiar one. Another application is given in the Annex where the differences between tax deductions and tax credit are demonstrated.

A. <u>The Ghanaian Tax System</u>

24. According to the Ghanaian tax rules, equation (12) may be rewritten as follows:

$$T = g_i \cdot (B - s) - k_i \qquad (15)$$

where B = basic salary

s = standard deduction or personal relief under the tax schedule

g_i = marginal tax rate applicable to B

$$k_i = \sum_{j=1}^{i-1} (g_i - g_j) \cdot B_j \quad \text{for } i > 1$$

$= 0$ otherwise

The values of g's and k_i's corresponding to the 1988 tax schedule are given in Table 1.

Table 1
Parametric Constants
Ghanaian Personal Income Taxation

Index	Marginal Tax Rate (%) (m_i)	Size of Bracket (cedi) (B_i)	Additive Constant (cedi) (k_i)
1	5	30,000	0
2	10	30,000	1,500
3	20	225,000	7,500
4	30	225,000	36,000
5	40	225,000	87,000
6	50	225,000	160,000
7	55	N.A.	208,000

Notes: 1. Definitions are given in the text.

Sources: 1. 1988 Budget Statement, Ministry of Finance
2. Staff calculations

25. To compare the Ghanaian tax schedule to that of other countries, it is desirable to convert the tax base from basic salary to gross salary inclusive of all allowances. The conversion is aided by the relation:

$$B = -[\ \gamma(1 + \delta)\ ] + [1/(1 + \delta)] \cdot Y \qquad (16)$$

Upon substitution of B from (19) , equation (18) becomes:

$$T = [g_i/(1 + \delta)] \cdot [Y - \gamma - s \cdot (1 + \delta)] - k_i \qquad (17)$$

Equation (17) deserves a closer look as it turns out to be of critical importance. Note in particular that equation (17) shows exactly the same relationship between tax liability and gross income as in equation (12), with the following notational changes:

$$m_i = g_i/(1 + \delta) \qquad (18)$$

$$x = \gamma + s\,(1 + \delta) \qquad (19)$$

$$k_i = \sum_{j=1}^{i-1}[(g_i - g_j)/(1 + \delta)] \cdot B_j(1 + \delta) \qquad (20)$$

This correspondence suggests that the Ghanaian tax system is equivalent to a conventional tax system (based on gross income and standard deductions) with marginal tax rates given by identity (18), standard deduction as in (19) and tax brackets as in (20).

26. Table 2 lays out the full equivalent tax schedule. The conversion is based on the following relation between tax-free allowances and basic salaries:

$$A = 18{,}300 + 0.2.B \qquad (21)$$

It is evident that the converted structure is considerably more generous than what is suggested by the existing schedule. The standard deduction of the converted schedule is much larger, the brackets wider and the marginal rates lower. In particular, the top marginal rate is 45.8%, compared to the 55% in the existing schedule. The two structures are presented diagrammatically in Figure 4. The differences between the two would be more pronounced if the coefficient of B in (21) were raised, as with an inclusion of bonuses in the tax-free allowances.

27. The equivalence of the two tax schedules is demonstrated in Table 3. Two alternative ways of calculating a taxpayer's liabilities are given. The existing method based on basic salary is shown on the left, while the alternative method based on gross income and the converted tax schedule is shown on the right. The resulting tax liabilities are identical.

Table 2

Existing Ghanaian Tax Structure and its Equivalent

Existing Tax Schedule (Based on Partial Income)		Equivalent Tax Schedule (Based on Total Income)	
Tax Rate (%)	Bracket (cedi)	Tax Rate (%)	Bracket (cedi)
Nil	First 24,000	Nil	47,100
5	Next 30,000	4.167	36,000
10	Next 30,000	8.333	36,000
20	Next 225,000	16.667	270,000
30	Next 225,000	25.000	270,000
40	Next 225,000	33.333	270,000
50	Next 225,000	41.667	270,000
55	Over 984,000	45.833	over 1,199,100

Table 3

Ghana: Alternative Income-Tax Calculations

Taxpayer's Basic Salary = C 300,000
Tax-Free Allowances = C 78,000
Total Income = C 378,300

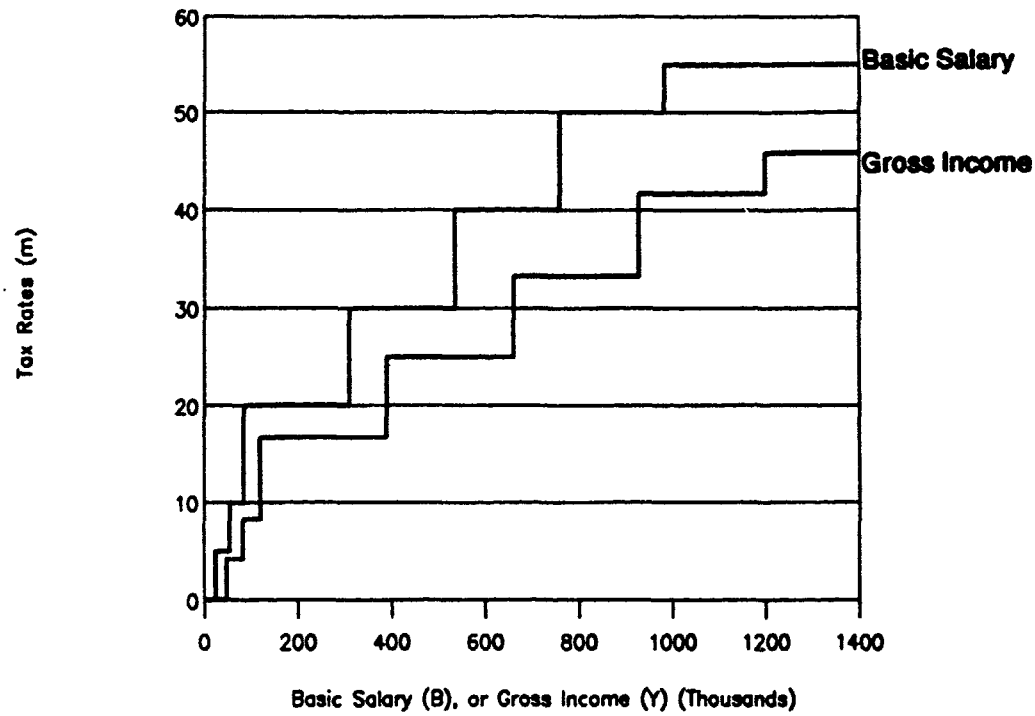
Tax Calculation

	Existing System[a]			Converted System[b]		
	Bracket	Rate	Tax	Bracket	Rate	Tax
	24,000	0.0	0	47,000	0.0	0
	30,000	0.05	1,500	36,000	4.167	1,500
	30,000	0.10	3,000	36,000	8.333	3,000
	216,000	0.20	43,200	259,200	16.667	43,200
Total	300,000		47,700	378,300		47,700

Notes: a. Based on basic salaries
b. Based on total income.

Figure 4

Ghana: Equivalent Tax Schedules



B. **The Nigerian Tax System**

28. The Nigerian tax is based on taxable income, according to the relation:

$$T = n_i . I - k_i \qquad (22)$$

where I = taxable income
n_i = marginal tax rates applicable I.

Moreover, from equation (11) above,

$$I = Y - D = -\alpha_N + (1-\beta_N) . Y \qquad (23)$$

Combining equations (22) and (23) gives:

$$T = n_i[(1-\beta_N) . Y - \alpha_N] - k_i \qquad (24)$$

The values of the parametric constants corresponding to the 1987 tax schedule are as follows:

α_N = 2,300 for a family of four;

β_N = 0.125

and the n_i's and k_i's are given in Table 4.

29. Like the Ghanaian tax system, Nigeria's can also be converted to an equivalent system based on total (or gross) income. The conversion rests on the observation that equation (24) may be rewritten as:

$$T = n_i(1-\beta_N) . [Y - (\alpha_N/(1-\beta_N))] - \sum_{j=1}^{i-1}(1-\beta_N)(n_i-n_j) . B_j /(1-\beta_N) \qquad (25)$$

Equation (25) is equivalent to equation (12) with the following change of notation:

$$m_i = n_i(1-\beta_N) \qquad (26)$$

$$x = \alpha_N /(1-\beta_N) \qquad (27)$$

Table 4
Parametric Constants
Nigerian Personal Income Taxation (1987)

Index	Marginal Tax Rate (%) (m_i)	Size of Bracket (Naira) (B_i)	Additive Constant (Naira) (k_i)
1	10	2,000	0
2	15	2,000	100
3	20	2,000	300
4	25	2,000	600
5	30	2,000	1,000
6	35	5,000	1,500
7	40	5,000	2,250
8	45	10,000	3,250
9	50	10,000	4,750
10	55	N.A.	6,750

Notes: 1. Definitions of constants are given in the text.
2. Deductions are N1000 plus 12.5 of earned income for the taxpayer; N 500 for spouse; N400 for each child and and N600 for each dependent relative.

Sources: 1. Federal Inland Revenue Department, Lagos.
2. Staff Calculations.

30. The resulting equivalent tax structures are shown in Table 5 and in Figure 5. Compared to the existing tax schedule, the converted schedule shows: (a) larger tax brackets; (b) lower marginal tax rates; (e) more liberal deductions. The equivalence of the two tax schedules is confirmed by the illustrative calculations given in Table 6.

Table 5

Nigeria: Comparison of Alternative Tax Schedules
(1987)

1987 Tax Schedule (Based on Taxable Income)			Equivalent Tax Schedule (Based on Total Income)		
Tax Rate (%)	Bracket (Naira)		Tax Rate (%)	Bracket (Naira)	
Nil	First	2,300	Nil	First	2628.6
10	Next	2,000	8.75	Next	2285.7
15	Next	2,000	13.125	Next	2285.7
20	Next	2,000	17.500	Next	2285.7
25	Next	2,000	21.875	Next	2285.7
30	Next	2,000	26,250	Next	2285.7
35	Next	5,000	30.625	Next	5714.3
40	Next	5,000	35.000	Next	5714.3
45	Next	10,000	39.375	Next	11,428.6
50	Next	10,000	43.750	Next	11,428.6
55	Over	42,300	48.125	over	48,342.9

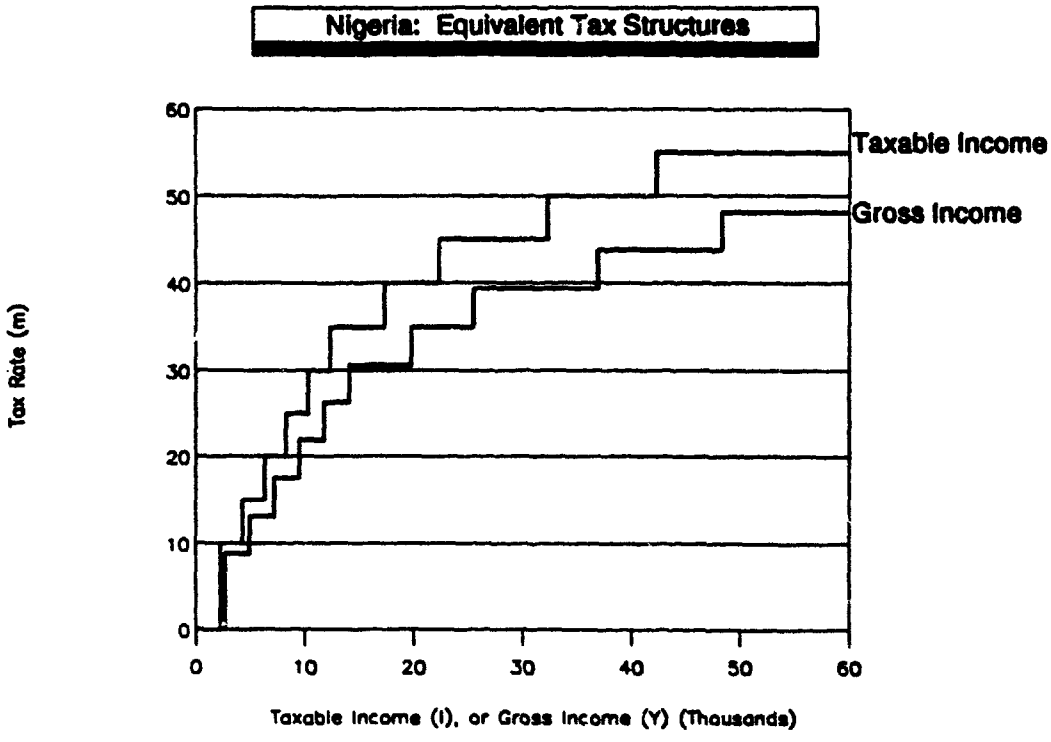
Table 6

Nigeria: Alternative Methods of Income Tax Calculation

Taxpayer's earned income	=	N 10,000
Dependent allowances	=	N 2,300
Relief for earned income	=	N 1,250
Taxable Income	=	N 6,450

	Actual Method (Based on Taxable Income)			Equivalent Method (Based on Total Income)		
	Bracket	Rate	Tax	Bracket	Rate	Tax
	2,000	0.10	200.0	2285.7	0.0875	200.0
	2,000	0.15	300.0	2285.7	0.1313	300.0
	2,000	0.20	400.0	2285.7	0.1750	400.0
	450	0.25	112.5	514.3	0.2188	112.5
Total	6,450		1012.5	10,000		1012.5

Figure 5


Nigeria: Equivalent Tax Structures
Taxable Income
Gross Income
Tax Rate (m)
60
50
40
30
20
10
0
0
10
20
30
40
50
60
Taxable Income (I), or Gross Income (Y) (Thousands)

IV. A MORE GENERAL FORMULATION

31. So far deductions have been expressed as a simple linear function of income. Such a function might be regarded as an approximation or a least-square fit to a more general relationship. In some cases, however, a simple linear function is not an adequate representation of the tax rules, and a different formulation is needed. A case in point is Nigeria's 1988 tax schedule, which differs considerably from the 1987 schedule considered above. When the relationship between deductions and income is general and possibly non-linear two questions arise:

(i) What are the conditions for ensuring that the tax system is progressive?

(ii) Can the tax system be represented by an equivalent conventional schedule?

32. Progressivity. A conventional tax system is progressive when the proportion of tax liability to income rises with income. Progressivity is generally achieved by the use of multiple and increasing marginal tax rates. A minimum of two marginal tax rates is required, involving a zero rate (as applied to personal relief and other allowances) and a positive rate. In practice, however, most tax systems apply a large number of marginal tax rates, with four or five rates being fairly common.

33. Progressivity (as measured by the rate of increase in the tax-income ratio) is relatively sharp before the top marginal tax rate is reached. Beyond that point, the same marginal tax rate applies throughout, and the tax-income ratio changes very gradually. At very high income levels, the average tax rate approaches the marginal tax rate and the two become virtually indistinguishable.

34. The conditions for progressivity may be stated as follows:

(a) $(dT/dY) > 0$; (28a)

(b) $(d^2 T/(dY)^2) > \text{or} = 0$ (28b)

where, as before, T = tax liability
Y = total income

Consider a general deduction function:

$$D = f(Y)$$

where D = tax deduction

f = a twice differentiable, but otherwise unrestricted non-linear function.

From equation (12) above, tax liabilities may be written as:

$$T = m_i (Y - f(Y)) - k_i \quad (29)$$

where m_i = marginal tax rate applicable to Y

and k_i = a constant determined by the tax schedule.

Thus, progressivity requires that:

from (28a) $(dT/dY) = m_i (1 - f') \geq 0$

$$\text{or } f' \leq 1; \quad (30a)$$

and from (28b) $(d^2T/(dY)^2 = -f'' \geq 0$

$$\text{or } f'' \leq 0 \quad (30b)$$

where f' = first derivative of function f
f'' = second derivative

35. The first condition states that the increase in deduction cannot exceed the increase in income. While true, it is nonetheless a trivial requirement as few would allow a provision that fails this test. The second condition provides more insight; it states that the slope of the

deduction function should be constant or falling (29b). In other words, to allow progressivity, deductions must be a constant or declining proportion of income. Figure 6 shows three general classes of functions and their consistency with progressivity. The tax system loses progressivity only when the deduction function is strictly convex or quasi-convex (with a line segment connecting two points on the curve lying entirely above or on the curve).

36. Representation. The analysis in Section III shows that a tax system with IDTDs can be represented by a conventional tax schedule, as long as the deduction function is linear. When the deduction function takes on a general form, however, there is no guarantee that such representation is possible. Consider equation (29) above:

$$T = m_i (Y - f(Y)) - k_i$$

Marginal tax rates for the system are given by:

$$(dT/dY) = m_i \cdot (1 - f'(Y)) \qquad (31)$$

which is in general a function of Y. To each real number Y corresponds a unique marginal tax rate which depends on Y. Since there are infinite values for Y, the marginal tax rates also take on infinite values. It is therefore not possible to represent such a tax system with a standard schedule.

37. There is, however, a class of functions that is more general than the straight lines and admits a conventional representation. It is the class of piecewise linear quasi-concave functions, as shown in Figure 7, which consist of two or more line segments. Piecewise linear deduction functions arise naturally when a tax system uses IDTDs that are subject

Figure 6

Deduction Functions and Progressivity

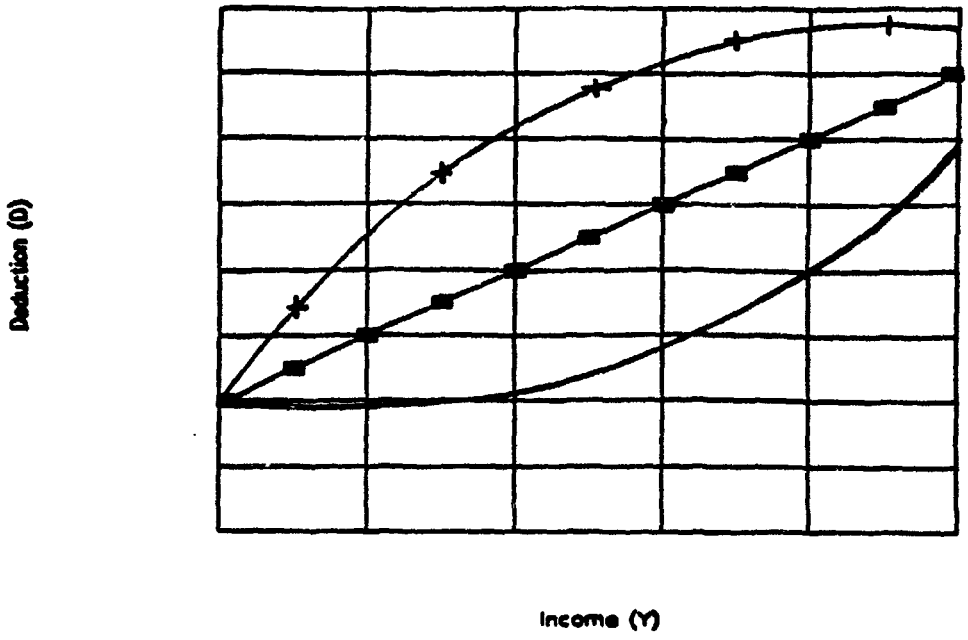
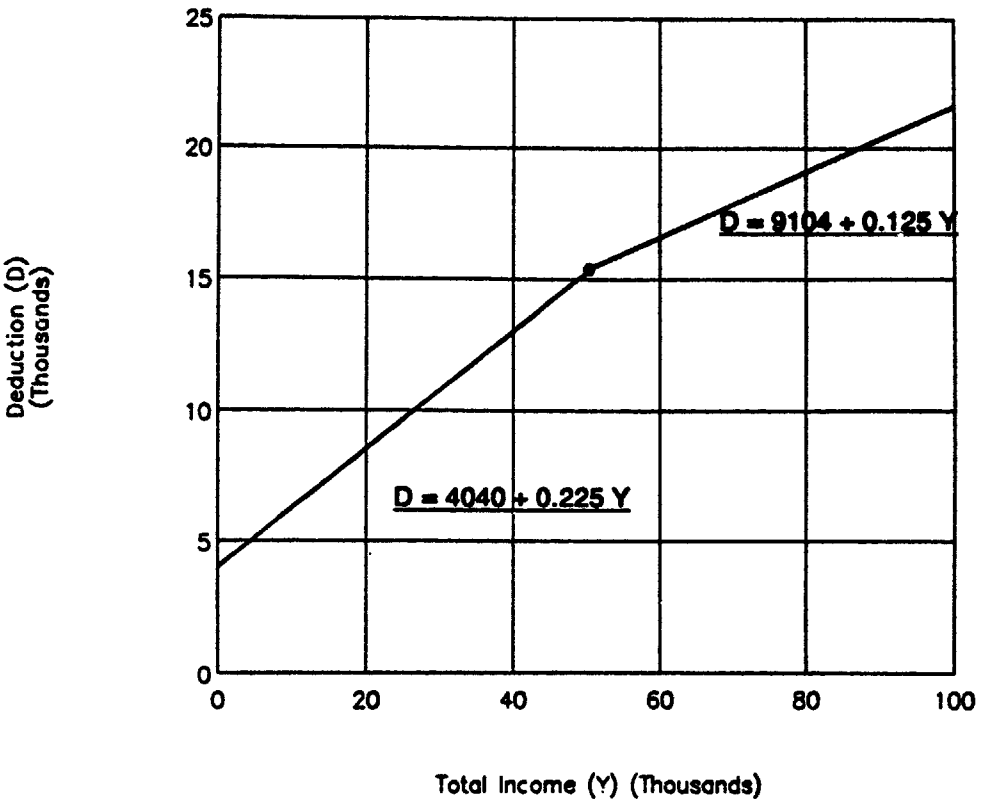


Figure 7

A Piecewise Linear Deduction Function (Based on Nigeria's 1988 Tax Schedule)



to a ceiling. (An example is given below) When the deduction function is piecewise linear, equation (29) may be written as:

$$T = m_i (Y - f_j (Y)) - k_{ij} \qquad (32)$$

where f_j = the jth linear deduction function within the ith bracket.

Marginal tax rates are given by:

$$(dT/dY) = m_i (1-\alpha_j) \qquad (33)$$

where $\alpha_j = f'_j$, a constant since f_j is linear.

From (33), it is clear that the number of marginal tax rate for the system is at the most:

$$r = q \cdot n$$

where q = number of original tax rates;
n = number of segments in the deduction function;
r = number of new tax rates,

which is finite. Representation by a conventional tax schedule is therefore possible.

38. The 1988 Nigerian Tax Schedule. The rates and brackets of the 1987 schedule, as shown in Section II above, were retained in 1988, but two changes in deductions were introduced. First, the transport allowance was raised from N600 to N1740 per annum. This change would merely shift the deduction schedule, as shown in Figure I, upward. Second, the housing allowance was now stated as 20% of basic salary, subject to a ceiling of N5064. This new rule added a new line segment to the origin schedule, as well as shifted it upward.

39. In practice, the components of compensation--basic salary, tax-free pay and allowance--are set independently. But there is regularity in the

proportions of these components. The basic salary is the largest item; tax-free pay ranges from one half to one quarter of the basic salary, while the combined allowances may vary from 50% to 100% of basic salary. For simplicity, the following analysis will assume that, for the relevant range of income, the housing allowance represents 10% of total income, subject to the stipulated ceiling.

40. With the above assumption, the deduction function may be written as:

$$D = 4040 + 0.225Y \qquad \text{for } Y \leq N50,640$$

$$D = 9104 + 0.125Y \qquad \text{for } Y > N50,640$$

where the coefficient of Y is the sum of housing allowance (10%) and income relief (12.5%). Of course, the percentage housing allowance becomes a flat allowance as income reaches N50,640.

This deduction function generates a quasi-convex, piece-wise linear schedule as given in Figure 7 above.

41. To convert the tax system to a more conventional one, the following changes are needed:

$$x \text{ (standard deduction)} = 4040/0.775 = 5212.9$$

$$m_i \text{ (marginal tax rate)} = n_i * 0.775$$

$$B_j \text{(tax brackets)} = B_j^N / 0.775$$

for income below N50,640.

Table 7

Nigeria: 1988 Actual and Equivalent Tax Schedules

1988 Tax Schedule (Base: Taxable Income)			Equivalent Tax Schedule (Base: Total Income)		
Tax Rate (%)	Bracket (Naira)		Tax Rate (%)	Bracket (Naira)	
Nil	First	4,040 a/	Nil	First	5,212.9
10	Next	2,000	7.75	Next	2,580.6
15	Next	2,000	11.625	Next	2,580.6
20	Next	2,000	15.5	Next	2,580.6
25	Next	2,000	19.375	Next	2,580.6
30	Next	2,000	23.25	Next	2,580.6
35	Next	5,000	27.125	Next	6,451.6
40	Next	5,000	31.00	Next	6,451.6
45	Next	10,000	34.875	Next	12,903.2
50	Next	10,000	38.75	Next	12,903.2
55	Over	44,040	42.625	Next	6,717.7 b/
			48.125	Over	50,640.0

Notes:

a. Includes personal deduction (N1,000), spouse allowance (N500), two children (N800) and transport allowance (N1740).

b. Beyond this point, the second part of the deduction rule applies with the housing allowance fixed at N5064 per annum.

For the rest of the taxpayers:

$$x = 9104/0.875 = 10,404.6$$

$$m_i = n_i * 0.875$$

$$B_j = B_j^N/0.875$$

Table 7 compares the 1988 tax schedule to the equivalent conventional one. Note that the ceiling on housing allowance takes effect only in the top income tax bracket; all other taxpayers are entitled to the proportional deduction.

V. CONCLUSIONS

42. At first sight, it seems dubious to let tax deduction increase with the taxpayer's income. Such a practice appears to give larger tax breaks to the rich than the poor. Upon a close examination, however, this notion turns out to be false. As the preceding analysis shows, each tax system with income-dependent tax deductions is fully equivalent to a particular conventional tax system with standard deductions.[9]

43. Consider a given conventional tax schedule that has standard deductions and progressive tax rates. Suppose to this system a new rule is added that provides an additional deduction equal to 10% of the taxpayer's income. This single reform measure has the same effects as a "liberalization" package consisting of: (a) an increase of about 10% in standard deductions; (b) an enlargement of about 10% in all tax brackets; (c) a reduction of about 10% on all marginal tax rates. The full equivalence of the two options may not be obvious to all observers; at least it was not to the authors.

44. An implication of the finding is that a tax system with IDTDs is not less equitable than a conventional one with STDs. They are in fact equivalent. Note in particular that a liberalization package of the above variety has been the main features of tax reforms across countries in the 1980s. Equity has not emerged as a major issue in these efforts. An equivalent tax reform option consisting of an IDTD rule, therefore, should not be a cause for particular concern. Observe, moreover, that removing an IDTD rule already in use and introducing a liberalization package at the

---

9/ Except when the deduction function is non-linear with strictly non-zero second derivatives, (as discussed in Section IV). Such a function has not been found in practice, however; the existing ones are either linear or piecewise-linear, with conventional representation.

same time would leave the tax system effectively unchanged, although on the surface the two sets of tax rules may appear to be very different.

45. The equivalence between an IDTD and a "liberalization" package as mentioned above also raises a further policy question. Should a country contemplating a tax reform to liberalize the tax schedule simply adopt an IDTD? There is no immediate or unconditional answer. Although an IDTD is an administratively simple procedure, its logic is not immediately transparent. It could be misinterpreted and regarded as inequitable. Moreover, it could be misused. The fixed percentage deduction could be relaxed, resulting in an unrestricted deduction as with the case of canteen and transport allowances in Ghana. In addition, the IDTD could be confined to one segment of taxpayers, resulting in a true inequity, as with the discrimination against the self-employed in Ghana. These caveats concerning IDTDs do not provide adequate justifications for replacing an existing IDTD by an equivalent liberalization package, however. A taxpaying public long accustomed to income-determined allowances may vigorously oppose such a change. Even in an imperfect system such as that of Ghana, it may still be strategically preferable to correct the aberrations, rather than eliminating the IDTDs and risking a tax revolt.

46. The finding also has implications for comparative tax research. When a country uses IDTDs, its tax schedule should not be compared directly to a conventional tax schedule in another country. Existing work dealing with cross-country comparisons of tax deductions and marginal tax rates generally fails to account for this important feature. As the analysis above makes clear, the presence of IDTDs invalidates a straightforward comparison. A transformation as suggested above is needed to put an IDTD system on a comparable basis with a conventional tax schedule.

ANNEX

Tax Deductions vs. Tax Credit

For a system relying on tax deductions, tax liabilities may be expressed, as in (12):

$$T = m_i \cdot (Y - x) - k_i$$

Consider an increase in personal deduction, x, which might occur annually. An increase in deduction reduces tax liabilities as follows:

$$dT = - m_i \cdot dx$$

Tax saving to a taxpayer is thus equal to the product of his marginal tax rate and the increase in deduction. Clearly, a taxpayer with high income receives a larger tax saving than one with low income.

Under a tax system with tax credit instead of deductions, the relationship (12) is modified somewhat:

$$T = m_i \cdot Y - c - k_i$$

where c = a specified tax credit.

An increase in tax credit gives rise to the following change in tax liabilities:

$$dT = -dc$$

Thus, regardless of income and marginal tax rates, tax saving is equal to the increase in tax credit. There is no variation across taxpayers. A numerical illustration of this point is given in Table A1.

Table A1

Standard Deduction vs. Tax Credit

I. Increase in Deduction

		Before		After	
		Mr. Green	Mr. Brown	Mr. Green	Mr. Brown
1.	Gross Income	20,000	40,000	20,000	40,000
2.	Tax	1,000	6,000	800	5,400
3.	Tax Saving (Before-After)	200	600		

II. Increase in Tax Credit

		Before		After	
		Mr. Green	Mr. Brown	Mr. Green	Mr. Brown
4.	Gross Income	20,000	40,000	20,000	40,000
5.	Tax	2,000	7,000	2,000	7,000
6.	Tax Credit	1,000	1,000	1,200	1,200
7.	Net Tax (5-6)	1,000	6,000	800	5,800
8.	Tax Saving (Before-After)	200	200		

Assumptions

Standard Deduction			Tax Credit		
Tax Rate	Brackets (Before)	Brackets (After)	Tax Rate	Brackets (Credit=1000)	Brackets (Credit=1200)
Nil	First 10,000	First 12,000			
10%	Next 10,000	Next 10,000	10%	First 20,000	First 20,000
20%	Next 10,000	Next 10,000	20%	Next 10,000	Next 10,000
30%	above 30,000	above 30,000	30%	above 30,000	above 30,000

BIBLIOGRAPHY

Lewis, S.R., Jr. Taxation for Development: Principles and Applications. Oxford University Press, 1984.

Shalizi, Zmarak, and L. Squire. "Tax Policy in Sub-Saharan Africa: A Framework for Analysis". Policy and Research Series Paper No. 2. Washington, D.C.: World Bank, December 1988.

Sicat, G.P., and A. Virmani. "Personal Income Taxes in Developing Countries". World Bank Economic Review 2 (1988), pp. 123-38.

U.S. Treasury Department Department. Tax Reform for Fairness, Simplicity and Economic Growth. Washington, D.C., 1984.

World Bank. World Development Report, 1988. Chapter 4, pp. 79-101. Washington, D.C., 1988.

PPR Working Paper Series

	Title	Author	Date	Contact
WPS176	Credit Rationing, Tenancy, Productivity, and the Dynamics of Inequality	Avishay Braverman Joseph E. Stiglitz		
WPS177	Cash-Flow or Income? The Choice of Base for Company Taxation	Jack M. Mintz Jesus Seade	April 1989	A. Bhalla 60359
WPS178	Tax Holidays and Investment	Jack M. Mintz	April 1989	A. Bhalla 60359
WPS179	Public Sector Pricing in a Fiscal Context	Christopher Heady	April 1989	A. Bhalla 60359
WPS180	Structural Changes in Metals Consumption: Evidence from U.S. Data	Boum-Jong Choe	April 1989	S. Lipscomb 33718
WPS181	Public Finance, Trade and Development: What Have We Learned?	Johannes F. Linn Deborah L. Wetzel	April 1989	M. Colinet 33490
WPS182	The Experience of Latin America With Export Subsidies	Julio Nogues	April 1989	S. Torrivos 33709
WPS183	Private Investment in Mexico: An Empirical Analysis	Alberto R. Musalem	April 1989	L. Spear 30081
WPS184	Women and Forestry: Operational Issues	Augusta Molnar Gotz Schreiber	May 1989	M. Villar 33752
WPS185	Uniform Trade Taxes, Devaluation, and the Real Exchange Rate: A Theoretical Analysis	Stephen A. O'Connell	April 1989	A. Oropesa 61758
WPS186	The Uruguay Negotiations on Subsidies and Countervailing Measures: Past and Future Constraints	Patrick A. Messerlin	April 1989	S. Torrijos 33709
WPS187	The Flexibility of Wages When the Output Price Changes: An Empirical Study of 13 Industrial Countries*	Menahem Prywes	April 1989	J. Israel 31285

PPR Working Paper Series

	Title	Author	Date	Contact
WPS188	International Comparisons of Wage and Non-Wage Costs of Labor	Luis A. Riveros	April 1	R. Luz 61762
WPS189	The Treatment of Companies under Cash Flow Taxes: Some Administrative, Transitional, and International Issues	Emil M. Sunley		
WPS190	Macro Performance Under Adjustment Lending	Riccardo Faini Jaime de Melo Abdel Senhadji-Semlali Julie Stanton	April 1989	M. Ameal 61466
WPS191	Openness, Outward Orientation, Trade Liberalization and Economic Performance in Developing Countries	Sebastian Edwards		
WPS192	Inflation, Price Controls and Fiscal Adjustment: The Case of Zimbabwe	Ajay Chhibber Joaquin Cottani Reza Firuzabadi Michael Walton	April 1989	A. Bhalla 60359
WPS193	Bank Lending to Developing Countries in the 1980s: An Empirical Test of Major Hypotheses on Voluntary and Involuntary Lending	Peter Nunnenkamp		
WPS194	Efficient Debt Reduction	Jeffrey Sachs		
WPS195	Commercial Bank Claims on Developing Countries: How Have Banks Been Affected?	Harry Huizinga		
WPS196	A Review of Alternative Debt Proposals	Eugene L. Versluysen		
WPS197	Cyclical and Secular Income Elasticities of Import Demand for Developing Countries	Fernando Clavijo Riccardo Faini		
WPS198	Equity in Unequal Deductions: Implications of Income Tax Rules in Ghana and Nigeria	Chad Leechor Robert Warner	May 1989	A. Bhalla 60359
WPS199	Private Sector Assessment: A Pilot Exercise in Ghana	Samuel Paul	May 1989	E. Madrona 61712